

LIGHTS OUT LINEAR ALGEBRA

A mathematical derivation on how to solve the Lights Out puzzle using linear algebra - with an implementation in JavaScript.

The first case is impossible since each button must have an odd number of neighbours in the solution for it to be unlit, and this contradicts lemma 1. Therefore at least one person shakes hands an even number of times. Therefore all button patterns in an unsolvable game change an even number of lights. The Toggle game. It is always possible to reach B without any intermediate position having all lights on or all lights off. It follows from this test that the only lights that can be switched on or off independently of the others are those that are not in any of the quiet patterns. Nevertheless, I'll keep that proof on the page, if only because it gives an explicit method for moving a light around the board. Swap the two equal rows. Let l be the button that was pressed to light up n so it is a neighbour of n , and is currently unlit. Suppose you push all the buttons. Some patterns have no solutions. If they have different effects on the lights, then we might be able to avoid the all-on or all-off position by replacing mn by nm . This is fairly trivial. Press a_k, a_{k-1} , The numbers of solutions ignoring rotation and reflection for $n=2$, It relies on the following lemma. Case 1: n is a neighbour of b , but not of c . Every light will now have changed $n-1$ times. This makes things a little more complicated. The move sequence $nbn a fbf$ is a lit-only equivalent to pressing a . Therefore the determinant is 0. The change in state of these neighbours will therefore change the parity of the number of lit buttons as well as the number of unlit solution buttons, and also the parity of the number of neighbouring pairs. Additionally, all four horizontally and vertically adjacent lights - forming a cross - get inverted as well when a single light gets pressed. If you have three adjacent buttons a,b,c c is not adjacent to a all unlit then the sequence $acbacb$ also involves four more unlit presses than lit ones.